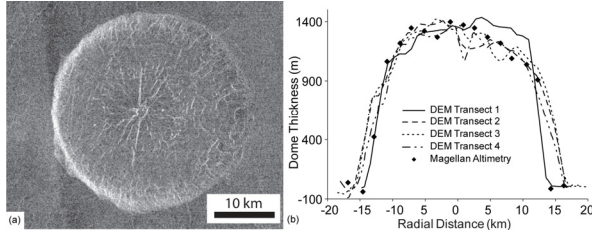


**Introduction:** One key to understanding the history of resurfacing on Venus is better constraints on the emplacement timescales for the range of volcanic features visible on the surface. Figure 1 shows a Magellan radar image and topography for a putative lava dome on Venus. 175 such domes have been identified with diameters ranging from 19 – 94 km, and estimated thicknesses as great as 4 km [1-2]. These domes are thought to be volcanic in origin [3] and to have formed by the flow of viscous fluid (i.e., lava) on the surface.



**Figure 1.** (a) Magellan image of a typical steep-sided dome in the Rusalka Planitia at 3°S, 151°E. (b) Topographic data for the dome shown in (a) with ~20x vertical exaggeration. The four transects depict topography from a digital elevation model generated from stereo Magellan images [4].

Fundamental issues surrounding the Venus steep-sided domes are the nature of their emplacement (e.g., duration), their composition, and the rheology of the lavas. A significant conundrum still persists: higher viscosity lavas are implied by the need to sustain the extremely thick flows (1 – 4 km) (e.g., [3]). However lower viscosity lavas are needed to provide the relatively “smooth” upper surface (e.g., [2]). There are also numerous quantitative issues that have implications for sub-surface magma ascent and local surface stress conditions. These include the nature and duration of lava supply, how long the conduit remained open and capable of supplying lava, the volumetric flow rate, and the role of rigid crust in influencing flow and final morphology.

Here we investigate a variety of scenarios for the emplacement of volcanic domes. Each scenario explores the effect of different boundary conditions on the solution of the Boussinesq equation for pressure-driven fluid flow in a cylindrical geometry:

$$\frac{\partial h}{\partial t} - \frac{g}{3\nu} \frac{1}{r} \frac{\partial}{\partial r} \left( r h^3 \frac{\partial h}{\partial r} \right) = 0 \quad (1)$$

One scenario assumes a constant volume of material, as has been proposed by previous investigators [5-8]. In this case, the dome thickness boundary condition at the source decays with time. We have found an exact analytical solution of (1) for this boundary condition based on an extended similarity analysis. Our solution removes the nonphysical singularities present in previous studies of volcanic domes on Earth and Venus [6-8]. A second scenario, just submitted for publication [9], allows a constant volume flow rate at the source of the dome. The approach used by [9] to solve (1) employs a combination of similarity analysis and singular perturbation theory. A final scenario revisits the concept of a viscosity that changes only with time, rather than with distance from the source, as suggested by [8].

**Fixed Volume Scenario:** One approach to modeling the viscous expansion of a dome is the assumption that most of the volume is emplaced rapidly, supply terminates, and the dome is formed by subsequent radial relaxation [5-8]. Results in [5-8] have used a similarity solution with a singularity at the origin, which also results in an infinite flow depth at the origin. Because (1) is based on lubrication theory (small Reynolds’s number and  $h/r$ ), the validity of such a solution is dubious.

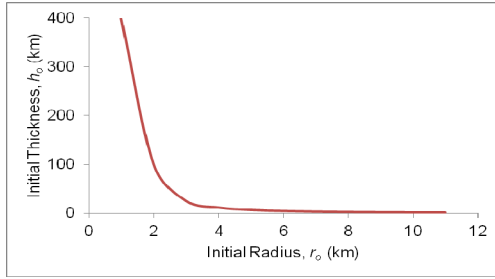
We have found an alternative to the approach by [5] (upon which [6-8] are based) to solving (1) that removes the singularity in the similarity solution by taking advantage of the translational invariance of (1) with respect to time (e.g., [13-14]). We have thus obtained the solution for flow thickness,

$$h(r, t) = \frac{h_o}{(1 + t/\tau)^{1/4}} \left[ 1 - \frac{1}{(1 + t/\tau)^{1/4}} \frac{r^2}{r_o^2} \right]^{1/3} \quad (2)$$

where the time constant,  $\tau$ , is given by

$$\tau = \frac{\nu}{4g} \left( \frac{3r_o^2}{4} \right)^4 \left( \frac{\pi}{V} \right)^3 \quad (3)$$

The thickness profile in (2) is finite for all values of  $r$  and  $t$ . However, because this boundary condition assumes all the material is already present on the surface, finding physically plausible values for both  $r_o$  and  $h_o$  (radius and thickness of dome when relaxation “begins”) is a challenge (Figure 2). Even with an  $r_o$  of 5 km, half the final radius, the height at the dome center would have to be 4 km which strains plausibility. With relaxation starting when  $r_o$  is 2 km, the height of the



**Figure 2.** Initial  $h_o$  as a function of  $r_o$  for a dome that relaxes to a final radius of 10 km and thickness of 1 km.

dome for this mode of emplacement would have to be 25 km which is clearly implausible. So, at best, this emplacement scenario might only apply to the very last stage of emplacement. Once this mode is reached, the flow front,  $r_f$ , advances at a rate that is unexpected for a diffusion-like process described by equation (1):

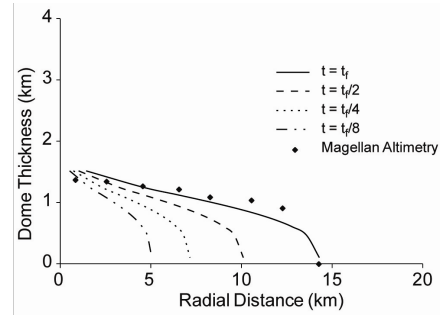
$$r_f(t) = r_o \left(1 + t/\tau\right)^{1/8} \quad (4)$$

From the definition of  $\tau$  in (3), dynamic bulk viscosities on the order of  $10^{16}$  -  $10^{17}$  Pa-s cannot be rigorously precluded, and suggest that this approach will over predict the viscosity and under predict the emplacement time. Note that the duration of lava supply cannot be estimated by this method because the pre-relaxation phase is ignored. Application of this approach (with the corrected math) to the dimension of Venus domes (e.g., Figure 1) suggests viscosities comparable to rhyolites. Even with the removal of the singularity, it is difficult to favor any inferences based on the decaying thickness boundary condition. It remains to be determined whether some combination of space and time viscosity changes can result in a viable emplacement scenario for this boundary condition.

**Constant Flow Rate Scenario.** The alternative is to assume the dome is fed continuously for the majority of the emplacement time. Unfortunately no analytic similarity solution has yet been found for this boundary condition and the numerical integration of the resulting ordinary differential equation is questionable. The similarity variable  $\eta = r^2/t$  is an obvious choice, producing the radial expansion of a dome proportional to  $t^{1/2}$ , as expected from the structure of (1).

However, as reported in [9], a combination of singular perturbation theory and similarity analysis provides a uniformly valid and accurate solution of (1) for a cylindrically symmetric flow that is fed continuously at a constant rate. The approach is based on a suite of analytic and numerical studies [10-12] not commonly referenced in the volcanologic literature. The volumetric flowrate used in [9] depends on an arbitrary power of the flow depth, making it possible to distinguish between Newtonian and other rheologies describing

terrestrial and planetary mass flows. The flow depth profiles (Figure 3) are shown to thicken as the front advances. Emplacement times are intimately correlated with the bulk rheology. Detailed fitting of the theoretical profiles to the shape of a typical dome on Venus indicates a bulk dynamic viscosity of  $10^{12}$  -  $10^{13}$  Pa-s and emplacement times of approximately 2 - 16 years, both consistent with basaltic andesite composition and both significantly less than prior estimates.



**Figure 3.** Axially symmetric Newtonian fluid flow profiles at four times and altimetry from Figure 1b.

**Conclusions:** The only emplacement scenario that appears credible at the present state of understanding of the steep-sided domes on Venus is one that involves the constant supply of lava for a period of years to a decade or so. The dimensions of these domes also suggests a composition that is more consistent with basaltic andesite, rather than more silicic compositions as has been previously suggested. No modeling to date has shown that significant periods of waxing or waning in the flow rate are reflected by the topography, dimensions, and morphology of the Venus domes. These results have significant implications for subsurface conditions and processes that can sustain such conditions.

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